S520 Homework 8

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10.5.#A-2: Although the data from this experiment is fundamentally discrete, I am going to treat it as continuos. To build a confidence interval with confidence 0.95, the following needs to hold: $1-\alpha = 0.95 \Longrightarrow \alpha/2 = 0.025$. The R command $qbinom(0.025, 20, .5)$ returns $k = 6$. By experimentation we obtain: $1 - pbinom(6, 20, .5) \approx 0.94$. Any other choice will be way off the value and thus, we can construct a confidence interval of 94%, very close to the required 95%.
The form of the interval is (sorting the values): $(x_{(k+1)}, x_{(n-k)}) = (x_7, x_{14}) = (239, 251)$

10.5.#C-4: $H_0 : \theta \leq 0$ vs. $H_1 : \theta > 0$, $\alpha = 0.05$

(a) Let $D_i = X_i - \theta_0 = X_i - 0 = \vec{d} = c(6.1, -8.4, 1, 2, 0.7, 2.9, 3.5, 5.1, 1.8, 3.6, 7.0, 3.0, 9.3, 7.5, -6.0)$.
The following table summirizes the neccesary information of the values D_i for the Wilcoxon test:

values	abs(values)	ordered(abs(values))	R_i	Positive Ranks	Negative Ranks
6.1	6.1	0.7	1	1	
-8.4	8.4	1	2	2	
1	1	1.8	3	3	
2	2	2	4	4	
0.7	0.7	2.9	5	5	
2.9	2.9	3	6	6	
3.5	3.5	3.5	7	7	
5.1	5.1	3.6	8	8	
1.8	1.8	5.1	9	9	
3.6	3.6	6	10		10
7.0	7.0	6.1	11	11	
3.0	3.0	7	12	12	
9.3	9.3	7.5	13	13	
7.5	7.5	8.4	14		14
-6.0	6.0	9.3	15	15	
		Sum:		96	24

Thus, $t_+ = 96$ and $W1.p.sim(15, 96) \approx 0.043/2$(two tails to one tail) $< 0.05 = \alpha \Longrightarrow$ reject H_0

(b) $W1.walsh(x) = 3.125$

(c) $W1.ci(x, .1) =$ [1,] 30 0.85 5.00 0.905 [2,] 31 1.00 4.95 0.899 [3,] 32 1.25 4.85 0.902 [4,] 33 1.35 4.80 0.867 [5,] 34 1.40 4.75 0.863. The estimated confidence coefficient for $k = 32$ is nearest to 0.90, so the desired ci is $(1.25, 4.85)$

10.5.#C-5:

(a) We want to use the sign test for the following hypothesis: $H_0 : \theta \leq 0$ vs. $H_1 : \theta > 0$, $\alpha = 0.05$.
From the data: $n = 15, y = \#\{x_i > 0\} = 13,$ and $c = min(13, 15 - 13) = 2$, so

$$\mathbf{p} = P(Y \geq y) = P(Y \geq 13) = 1 - P(Y \leq 12) = 1 - pbinom(12, 15, .5) = 0.0034 < 0.05 \Longrightarrow reject H_0$$

(b) The sample has an odd number of values, i.e. $n = 2m+1 = 15 \Longrightarrow m+1 = 8$, so if we order the values in ascending order, the median is $x_8 = 3.0$.

(c) To build a confidence interval with confidence 0.90, the following needs to hold: $1 - \alpha = 0.90 \Longrightarrow \alpha/2 = 0.05$. The R command $qbinom(0.05, 15, .5)$ returns $k = 4$. By experimentation we obtain: $1 - pbinom(4, 15, .5) \approx 0.94$. Any other choice will be way off the value and thus, we can construct a confidence interval of 94%, very close to the required 95%.
The form of the interval is (sorting the values): $(x_{(k+1)}, x_{(n-k)}) = (x_5, x_{11}) = (1.8, 5.1)$

10.5.#D-3: To build a confidence interval with confidence 0.90, the following needs to hold: $1-\alpha = 0.90 \Longrightarrow \alpha/2 = 0.05$. The R command $qbinom(0.05, 20, .5)$ returns $k = 6$. By experimentation we obtain: $1 - pbinom(6, 20, .5) \approx 0.94$. Any other choice will be way off the value and thus, we can construct a confidence interval of 94%, very close to the required 95%.
The form of the interval is (sorting the values): $(x_{(k+1)}, x_{(n-k)}) = (x_7, x_{14}) = (0.609, 0.670)$